MATHEMATICS TEACHING

THE BULLETIN OF THE

ASSOCIATION for TEACHING AIDS in MATHEMATICS

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THE BULLETIN OF THE
ASSOCIATION FOR TEACHING AIDS IN MATHEMATICS

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No. 2 MAY, 1956 3/-

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EDITORIAL

Our first task is to acknowledge the many expressions of goodwill, both from this country and other parts of the world, which greeted our first number to appear in print.

The advent of mechanical computing on a large scale is bringing about rapid changes in mathematical practice in industry and commerce, and it is part of a teacher's job to keep pace with these developments, both for his own general education and also in order that he may teach the mathematics which the world is actually using. We hope to publish a series of articles on these modern developments, and the first appears in this number. Mr. R. A. Fairthorne, who writes with inside knowledge of Government research, has encouraged the work of this association since its early days, his ideas are often ahead of their time and he always expresses them in a controversially forthright manner.

Two issues ago we expressed a wish to publish material of interest to teachers in Primary and in Modern Schools. The problems in these schools are difficult and the need for effective teaching aids is very urgent, but we did not know that ideas would be so hard to get. How should mathematics be taught in Modern Schools? The people who can best say are those who actually teach there, but they are most reticent about the work they do. The more elementary the level of mathematics the more important is good teaching. This is a forum in which these questions can be discussed and we are most anxious to pool the experience which is available.

THE PAPER CURTAIN

It is part of our work to investigate teaching material which is made in other parts of the world, but the difficulties which we have to face do not always arise from language or distance. Witness the case of the film *Non-Euclidean Geometry* which recently arrived in this country and is reviewed later in this bulletin.

Having read of this film we wrote to the authorities in East Berlin sending only a simple enquiry (on unheaded notepaper). Before long we heard that the Zentral Institut für Lehrmittel was sending us a copy to view, and they were prepared to lend it to us for an indefinite period. The film came through the Iron Curtain easily enough, and in due course arrived in England. Then our difficulties started.

We were notified that the film was in the Post Office Customs, and we were requested to fill up the four pages of Form C160 which was enclosed and to apply to the Board of Trade for an Import Licence. This required a letter, a further six pages of form filling and a few days' wait; but even then the hunt had only just begun. The film could have been used immediately at a University lecture, but after travelling across London expecting to collect it we were told that we could not even view it privately unless we paid £3 18s. 5d. duty. Advice from educationalists with experience of these things is that whilst refunds can theoretically be made it is a safer course to keep your money, rather than pay and try to get it refunded later.

So the next step was to get our film without paying a ransom. This involved a further letter to a different branch of the Board of Trade, another three or four pages of forms and a wait of three weeks. A Duty Free Licence arrived in due course and our hopes ran high as we forwarded what was by now a bundle of forms to the Post Office ; but before long they were all back again with two copies of yet another form on which the instructions conflicted in a minor respect with the instructions on one of the earlier forms from the Board of Trade. Hoping that this would not disqualify us entirely we filled these forms up as well, and eventually we got our film—more than two months after it had been received in the country.

The balance sheet is as follows. The owners of the film, an official organisation of another country, were prepared to lend it to teachers they did not even know merely on the receipt of a personal letter; but the film could only be accepted nor the clerical assistance to cope with bureaucratic absurdities of this kind. A in this country at the cost of six letters, six telephone calls, a journey across London, sixteen pages of form filling and two months of wasted time.

This type of thing makes us look ridiculous abroad, and we must protest against it in the strongest possible terms. Practising teachers have neither the time customs official made the suggestion, quite seriously, that delay could be reduced by sending students on messages across London to collect forms. A strange conception of the function of technical education!

A scholar who publishes a book or paper can send it across international boundaries with considerable freedom. Scientific and educational films fulfil the same function as technical papers, but international barriers apply to films which do not apply to books. The International Scientific Film Association and Unesco have sought to have customs barriers on films lifted, but the obstacles still remain; and I.S.F.A. has found it necessary to publish what Mr. John Maddison describes as "a concise and sensible memorandum on how to approach importation problems —and customs officials"!

It may be argued that there is no ground for complaint when a procedure by which educational films can come in duty free does exist. But a teacher doing research work should not need to be an expert on import procedure, and the procedure should not involve sixteen pages of forms and two months' delay. Even if the day of freedom for the film is still far off the present procedure could be greatly simplified. It is difficult to see the need to deal with three quite separate branches of the Civil Service when one would do, and it should not be necessary to fill up four forms when one would do, and when many of the forms are clearly designed for large scale commercial transactions of a totally different nature.

Apart from the personal inconvenience caused the present procedure is discourteous to those in other countries who are trying to help us, and it holds British institutions up to ridicule and contempt.

SNOOKERED?

Snooker balls lie on a table illuminated by a single fixed point source of light. Their shadows on the table are ellipses which vary in area. The problem is to demonstrate in an elegant manner that the minor axes of these shadows are all of the same length.

E. HOGBEN.

APPARATUS TO ILLUSTRATE FOURIER SERIES

F. MERRY

Simple apparatus to illustrate the common formulæ connected with Fourier series may be constructed by anyone possessing a little mechanical skill and plenty of patience!

. The idea behind the apparatus is to construct a number of sine waveforms—three or four are usually enough—using milk-straws cut to required lengths for the ordinates. These are threaded on to wires and are held in place by "combs". When the combs are removed the straws fall down, adding their lengths along each wire to give the summation required.

The following are some of the Fourier series which have been constructed and demonstrated:—

1. $\sin \theta + \sin (\theta + 120°) + \sin (\theta + 240°) = 0$ (Three-phase system of electricity supply).
2. $\sin \theta + \frac{1}{9} \sin 3\theta + \frac{1}{25} \sin 5\theta + \ldots$ (Plucked string).
3. $\sin \theta + \frac{1}{3} \sin 3\theta + \frac{1}{5} \sin 5\theta + \ldots$ (Square wave).
4. $\sin \theta \pm \frac{1}{2} \sin 2\theta + \frac{1}{3} \sin 3\theta \pm \ldots$ (Saw-tooth wave).
5. "Beats"—frequency ratio 8 to 9.
6. Waves varying in frequency and/or amplitude.

A separate model is required for each series although the + or − of Number 4 is catered for on one model only, as is explained later in this article.

The materials required for the construction of any one board are as follows:—

*(*a*) Two pieces of $\frac{1}{2}''$ aluminium angle 16″ long each having 61 holes about $\frac{1}{8}''$ diameter spaced $\frac{1}{4}''$ apart along one side,† and about four holes along the other. (*A* in Fig. 1 and Fig. 3.)

*(*b*) Two pieces of aluminium strip 18″ × 1″ × 22 S.W.G. each having 61 holes spaced as above† along the centre, then slots cut out thus making two "combs". (*B* in Fig. 2 and Fig. 3.)

(*c*) Two strips of wood 18″ × $\frac{3}{4}''$ × $\frac{1}{8}''$ (Old T-squares) to give strength to the combs. (*C* in Fig. 2 and Fig. 3.)

(*d*) Four Terry Clips size 000. (*D* in Fig. 3.)

(*e*) Drawing board (hardboard, etc.) 23″ × 16″. (*E* in Fig. 1 and Fig. 3.)

(*f*) Two pieces of wood 16″ × $\frac{1}{2}''$ × $\frac{1}{2}''$, one with a hole $\frac{1}{2}''$ from each end to take a wood screw. (*F* in Fig. 3.)

(*g*) Welding rod or budgerigar cage wire about 14 S.W.G.—61 pieces each about 15″ long. (*G* in Fig. 3.)

(*h*) 61 + 61 + 61 coloured beads—three colours.

(*i*) Strip of plastic foam 16″ × $\frac{1}{2}''$ × $\frac{1}{2}''$.

(*j*) Screws, rivets, milk straws, etc.

(†The author uses a Prestacon punch—similar to a Junceto—for this purpose.)

*Alternatives to the $\frac{1}{2}''$ angle aluminium are suggested at the end of this article.

Screw the $\frac{1}{2}$" angle pieces to the board leaving about $\frac{3}{4}$" from the longer edge. (Fig. 1.)

Bend $\frac{1}{4}$" of one end of each wire at right angles making an L shape.

The straws are now tackled.

SQUARE WAVE

Suppose we wish to make an approximate Square Wave using the series $A \sin \theta + \frac{1}{3} A \sin 3\theta + \frac{1}{5} A \sin 5\theta + \ldots$

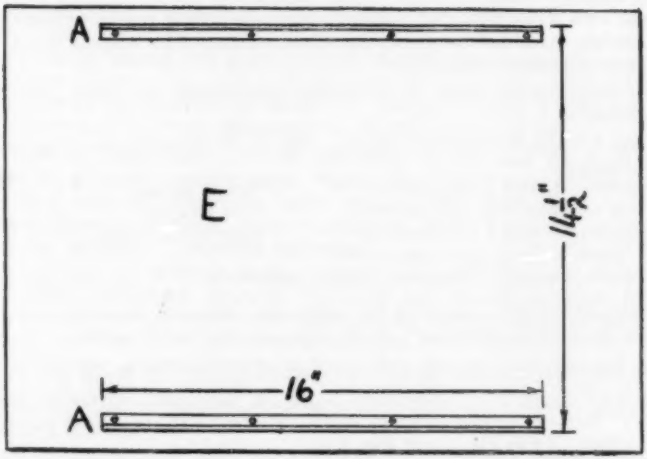

Fig. 1

Calculate $A \sin \theta$ every 6°, make $A = 3$" to get a decent amplitude (and to make the arithmetic easy!) and add on 4". This will make the longest straw 7" long ($\sin 90° \times 3" + 4"$) and the shortest straw 1" long ($\sin(-90°) \times 3" + 4"$).

It is convenient to make a table as under:—

TABLE 1 $A \sin \theta$ $(A = 3")$

θ	0°	6°	12°	18°	24°	30°	36°	42°	48°	54°	60°	66°	72°	78°	84°	90°
3 sin θ	·00	·31	·63	·93	1·22	1·50	1·77	2·01	2·23	2·43	2·60	2·74	2·85	2·93	2·98	3·00
Straw No.	1	2	30	3	29	4	28	5	27	6	26	7	25	8	24	9	23	10	22	11	21	12	20	13	19	14	18	15	17	16
Length (in.)	4·00	4·31	4·63	4·93	5·22	5·50	5·77	6·01	6·23	6·43	6·60	6·74	6·85	6·93	6·98	7·00
Straw No.	31	61	32	60	33	59	34	58	35	57	36	56	37	55	38	54	39	53	40	52	41	51	42	50	43	49	44	48	45	47	46
Length (in.)	4·00	3·69	3·37	3·07	2·78	2·50	2·23	1·99	1·77	1·57	1·40	1·26	1·15	1·07	1·02	1·00

If one can find straws 8″ long or over the cutting is simplified as:

Straw 1+ straw 31 are marked off together at 4″ and 8″.
Straw 2+ straw 32 are marked off together at 4·31″ and 8″.
Straw 3+ straw 33 are marked off together at 4·63″ and 8″.
and so on.

It is well to number each straw *before* cutting off with scissors.

These are now threaded on to a wire adding a coloured (say green) bead to each straw, and in this way $A \sin \theta$ is constructed.

Similarly $\frac{1}{3} A \sin 3\theta$ is constructed using the technique of cutting long straws with short ones, adding 2″ to each ordinate making the longest straw 3″ long and the shortest 1″ long (Since $\frac{1}{3} A = 1$″).

The table is as under:—

TABLE 2 $\frac{1}{3} A \sin 3\theta$ $(A = 3$″$)$

3θ	$0°$	$18°$	$36°$	$54°$	$72°$	$90°$
sin 3θ	·00	·31	·59	·81	·95	1·00
Straw	1	2 10	3 9	4 8	5 7	6
Number	21	22 30	23 29	24 28	25 27	26
	41	42 50	43 49	44 48	45 47	46
Length (in.) ..	2·00	2·31	2·59	2·81	2·95	3·00
Straw	11	12 20	13 19	14 18	15 17	16
Number	31	32 40	33 39	34 38	35 37	36
	51 61	52 60	53 59	54 58	55 57	56
Length (in.) ..	2·00	1·69	1·41	1·19	1·05	1·00

Straw 1+ Straw 11 are marked off together at 2″ and 4″.
Straw 2+ Straw 12 are marked off together at 2·31″ and 4″.
Straw 3+ Straw 13 are marked off together at 2·59″ and 4″.
and so on.

Straws 21 and 31, 22 and 32—41 and 51, 42 and 52—are cut in a similar way.

These straws again are numbered *before* cutting off and *using a different colour of ink*. These are threaded on to the corresponding wire followed by a bead of a colour different from the first—let us say blue.

Similarly $\frac{1}{5} A \sin 5\theta$ is constructed adding 2″ to each ordinate making the longest straw 2·60″ long and the shortest 1·40″ long.

Table 3 shows the lengths of each straw.

TABLE 3 $\frac{1}{6} A \sin 5\theta$ $(A = 3'')$

5θ ..	0°	30°	60°	90°
·6 sin 5θ ..	·00	·30	·52	·60
Straw ..	1 13 25 37 49	2 14 26 38 50	3 15 27 39 51	4 16 28 40 52
Number ..		6 18 30 42 54	5 17 29 41 53	
Length (in.)	2·00	2·30	2·52	2·60
Straw ..	7 19 31 43 55	8 20 32 44 56	9 21 33 45 57	10 22 34 46 58
Number ..	61	12 24 36 48 60	11 23 35 47 59	
Length (in.)	2·00	1·70	1·48	1·40

Here again several straws can be cut together.

Straw 1+ straw 7 are marked off at 2″ and 4″
„ 2+ „ 8 „ „ „ „ 2·30″ „ 4″
„ 3+ „ 9 „ „ „ „ 2·52″ „ 4″
and so on.

Number again in a third colour of ink *before* cutting, thread on to the corresponding wire and add a bead of another colour, say red.

Each wire should now have a straw, a green bead, a second straw, a blue bead, a third straw and a red bead.

Wire 1 is hooked into the first of the 61 holes of the first angle piece screwed to the board. The other end is pushed through the first hole of the second angle piece. Wire 2 is hooked into hole number 2 and the other end pushed into hole number 2 of the second angle piece.

In this way all the 61 wires are threaded.

Fasten immediately one of the 16″ × $\frac{1}{2}$″ × $\frac{1}{2}$″ pieces of wood in the $\frac{3}{4}$″ margin left between the $\frac{1}{4}$″ legs of the wires and the edge of the board, screwing it up as near to the angle piece as possible, though a little slack is not disastrous! The longer leg should protrude about $\frac{1}{4}$″ at the other end. (Fig. 3.)

Use Bostick Adhesive to stick the plastic foam strip to the other 16″ × $\frac{1}{2}$″ × $\frac{1}{2}$″ piece of wood. This is used to hold one set of straws in place when separating the various waves from the summation. This is not absolutely necessary but saves a lot of re-setting by hand.

Lay the board flat and tilt the bottom edge so that all the straws and beads slide to the top. Lay flat again and press the plastic foam strip on the top series of straws. Now tilt the top edge of the board and all the straws and beads except the top wave with its beads will fall to the bottom. Place a comb under this top wave and screw two Terry clips to hold it in place. (Fig. 3.)

Repeat the above process for the middle wave.

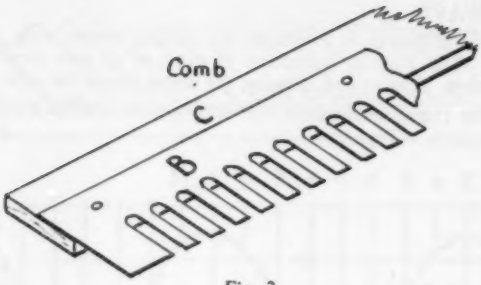

Fig. 2

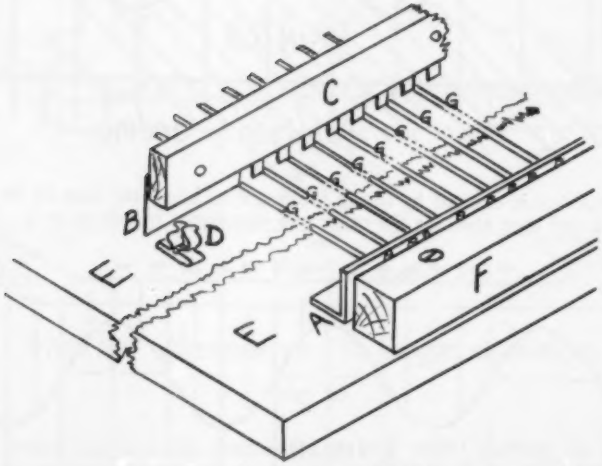

Fig. 3

Stand the board upright and the three separate waveforms will be indicated by the coloured beads. Remove the combs and the three waves are added giving the approximate square wave.

By using a larger board (or 16″ × 23″ the other way round), longer wires and more straws, further waveforms may be added showing how the resultant approaches more and more nearly to a square wave.

SAW-TOOTH WAVE

The + or − difficulty is overcome by cutting straws only for the sin θ and $\frac{1}{3}$ sin 3θ curves. The $\frac{1}{2}$ sin 2θ waveform is cut out of thin card and woven over and under the wires. The straws slide on the wires above the card.

By tilting the board backwards and sliding the card to Position 1 the values of $\frac{1}{2}$ sin 2θ are added. (Fig. 4A.)

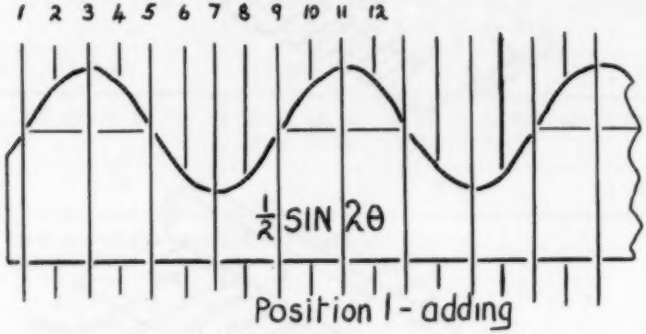

Fig. 4A

Subtraction is effected by tilting the board to allow the card to be pulled into Position 2 and then allowing the other two waveforms to fall on to it. (Fig. 4B.)

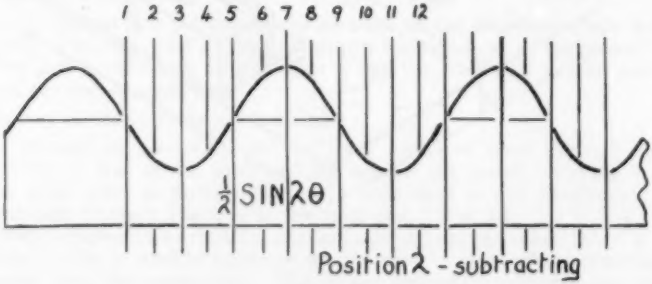

Fig. 4B

An alternative to angle aluminium and strip is to use corrugated cardboard, painted with melted wax to give it strength. Four strips are required, each with 61 or more hollows. These are glued to strips of wood and after being painted with wax are ready for use. The top and bottom strips have a further strip of wood screwed to them to hold the wires in place. (Fig. 5A.)

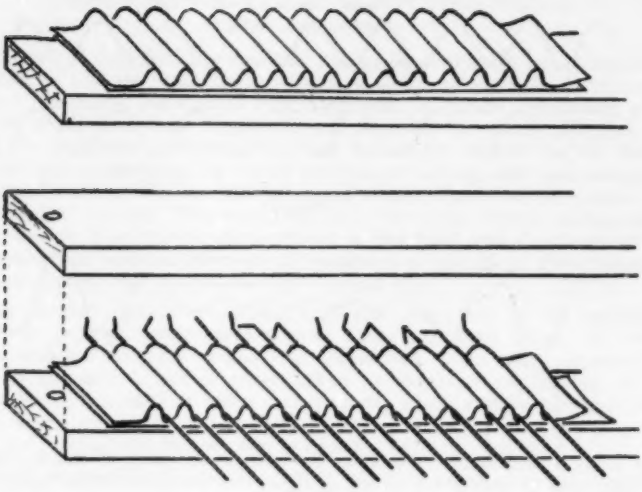

Fig. 5A

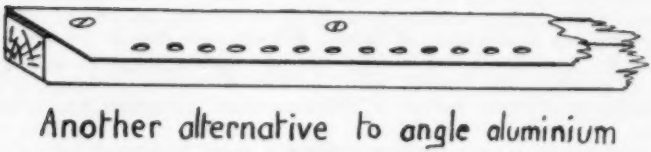

Fig. 5B

The author has used this material to make a model showing the addition of two sine waves with any desired phase difference (from 0° to 360°) and whose :—

(*a*) amplitudes are equal and frequences equal.
(*b*) „ „ „ but „ different.
(*c*) „ different „ „ equal.
(*d*) „ „ and „ different.

Another model which would be of use to the Physics teacher is one which shows how "*beats*" are caused. Two waveforms are cut—each having 73 straws—to show waves of frequencies 8 and 9 (Fig. 6). For this the straws are calculated for every 40° and 45° respectively. When these are allowed to "add" the resultant wave shows a "beat" in the middle. (Fig. 7.)

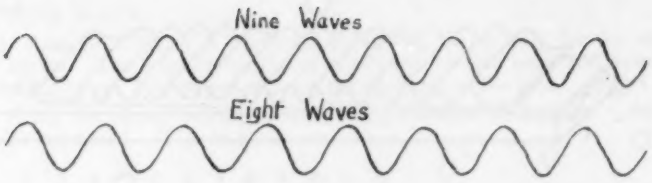

Fig. 6

Fig. 7

By using a cardboard wave of say 8 waves, in the place of straws, the position of the beat may be shifted to the left or right, this showing the increased amplitude to better effect. (Fig. 8.)

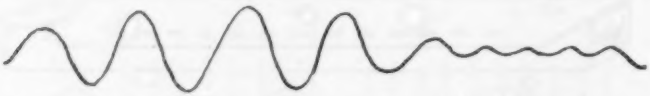

Fig. 8

By using waveforms of say 7 and 9 waves (64 wires) or 8 and 10 waves (81 wires) the formation of two beats can be shown.

The author will be pleased to give further details of the other waveforms mentioned to anyone interested.

The manufacture of a construction kit for this apparatus is being considered. Such a kit would include aluminium ready punched for use, combs, etc., and purchasers would be able to cut straws for themselves to make up models according to their own requirements. Enquiries can be addressed to the Editor.

COMPUTING WANTS AND COMPUTING NEEDS

ROBERT FAIRTHORNE

The advent of commercially available automatic computers has enlarged computing activities, and the intention to indulge in them, enormously, suddenly, and top-heavily. Inevitably, knowledge and experience of actual computing is very dilute at all levels, and there is general lack of ability to distinguish good work from bad. In particular, craftsmanship and knowledge needed on the humbler levels are at best unrecognised by senior or even co-ranking staff, and unasked for by recruiters. In short, whatever the rank and salary, almost all computers are at present amateurs together. This would be all to the good were it recognised, but humility is not yet a prominent characteristic in this field, and a widespread belief that computing was invented by electrical engineers round about 1944 means that relevant work before that date is literally a closed book.

At the moment, junior computing staff find themselves in an exacting job carrying little prestige, under seniors who know no more about the relevant techniques and are no more skilful than themselves. Often office organisation is bad, for an honours degree in mathematics or electrical engineering does not necessarily imply any skill at office management, which constitutes four-fifths of computing when run as a service. Further, such training as may be supplied is rarely, if ever, carried out by, or even in consultation with, teachers.

The school teacher can do much towards restoring sanity and has, indeed, the whip hand; for the essential raw material of administrative empire building is staff. The teacher should refuse to recommend institutions to potential recruits unless they are assured that their pupils' skill and competence will receive the same reward, respect and prestige as do those of engineering and science entrants; that their seniors, though compelled at present to short-circuit the irreplaceable qualification of being able to do the job themselves, will at least refrain from despising sub-graduate computing staff as "mere clerks" (those tending electronic equipment are not treated as "mere workmen"); and that teachers of mathematics will have some say in the teaching of mathematics in internal training schemes, especially at the elementary levels where the students are (as is inevitable when demand exceeds supply) neutral or only mildly interested about the subject. Finally, the teacher must never forget that, whatever the recruiting officer may know about the most modern heavy artillery of computation—electronic computers and all that— the teacher knows much more about computing itself, even though he has taught it under the excellent name of "Practical Mathematics." He, or she, certainly knows much more about junior computors as human beings which, as mechanisation grows, is their most important aspect, even when considered solely as computing agents.

With all this in mind we become less vulnerable to sales-talk, and can consider what computing institutions need; not what they ask for.

First, computing is clerical work. Basic requirements are neatness, precision, reliability, a temperament equable up to and including the phlegmatic (mercurial people will be miserable in every sense), and manipulative deftness. You cannot be both clumsy and a computor.

The indispensible mathematical skills are those already taught as Practical Mathematics; reasonable arithmetic fluency, intelligent use of graphs, tables and logarithms (cf. the Honours Mathematics graduate who asked me for a logarithm table with fifteen-figure *arguments*) with some idea of how and why they work; curve-tracing and plotting and, possibly, use of the slide rule, the latter almost entirely for its heuristic value (it has little application in a computing unit); and a thorough grounding in elementary algebra and algebraic techniques.

The last, in particular the Remainder Theorem and the handling of polynomials, is a basic requirement. Synthetic Division by linear and quadratic factors at least, must be taught as a manipulation as well as an intellectual process. This is the basic computing schema. From it comes the evaluation of partial fraction numerators by the Euclidean algorithm, the evaluation of the value of a polynomial and its derivatives and divided differences (and hence numerical solution of algebraic equations by improvement of approximate linear or, better, quadratic factors), and all realisable forms of interpolation.

The computational schema presenting the successive division of a polynomial by linear divisors until the quotient is a constant, together with the special case of the GCD algorithm, comprises the whole of polynomial computational operations. Given enough of the dividend, divisors, remainders and quotients we can reconstruct the rest of the schema. Different processes, such as Newton's expansion in terms of divided differences, Taylor's expansion (Newton with confluent divisors), evaluation of derivatives and differences and summation thereof, interpolation, Horner's scheme for change of variable and so on, are simply interpretations of the process of filling up the complete schedule given different initial data. All steps have immediate graphical interpretations for those who feel happier with them, and only after pupils have actually been doing "Finite Differences" need they be told so or have the Greek alphabet slung over their heads. "Upside down triangles" and similar exotic signs paralyse and scare off for ever many potentially useful computors if displayed too soon.

The fundamental importance of the iterated division schema, is shown by the fact that its theoretical interpretations are as significant as its computational applications. This is because it is essentially an existence theorem, and existence theorems must be also algorithms. Emphasis on this need not therefore sacrifice those who are not computors but latent algebraists. The equation solver now called the "Isograph," for instance, is actually and unintentionally a working model of a proof of the fundamental theorem of algebra. Similarly the Euclidean GCD algorithm is the existence theorem for unique partial fractions, and there is therefore no need for electrical engineers to find the numerators of partial fractions arising from "operational calculus" by solving simultaneous equations and splitting quadratic factors into complex conjugate linear ones. (Yes, they really do that!)

The last remark indicates why algebraic techniques are essential to the junior computor, quite apart from the fact that all computing as such must be algebraic, being finite in its entities and operations. (The passage from physics to analysis and from analysis to computation is a delicate, not to say dubious, matter which does not come within a junior's duties.) In order to fit calculus into science and

engineering something had to come out. The something was algebra beyond the Remainder Theorem. Thus the engineer dropped algebra to learn calculus, was then taught "operational" methods for turning his calculus into algebra, and then found himself without the algebraic knowledge needed to handle it. So it is quite common to find such people using Fourier and Laplace Transforms glibly and correctly, but evaluating polynomials by listing powers, multiplying and adding them and, if the derivative is needed, treating this as a new polynomial to be evaluated in the same way.

Linear algebra, in the sense of matrix arithmetic and elementary interpretation thereof, is essential up to a point. What point depends a good deal on what elementary text-books are available these days. When, in the mid-1930's, the author had to introduce matrix work to his computors, there were no elementary texts. It was largely a post-graduate topic. So it was shown that there were consistent and significant ways of adding and multiplying sets of numbers as well as single numbers; e.g. product of price-vector and goods vector was the amount of the bill. As it was a group concerned with aircraft structures, matrices could be interpreted as, and frequently were, sets of "influence coefficients," so latent vectors could be interpreted as the types of structural loading that deflected the structure into the same "shape" as themselves. The latent roots were then the proportionality factors. Similar interpretations are possible in terms of response to time series, and matrix equivalents to change of axes are valuable to those familiar with the geometrical ideas, though these days they may understand matrix equivalence better than they do the geometry.

I can confidently predict that abstract algebras in general, and lattice algebra in particular, will be key subjects in practical mathematics of the reasonably near future. They are not so much the algebras performed by computing, but the algebras needed to talk about computing. Boolean algebra, for instance, has now become fashionable amongst computing engineers, though those who introduced it into this field some years ago have long since found it inadequate except in restricted contexts. However, there is no reason why the ideas should not be displayed for the interest of those who may be interested. It is only the syllabus, not intrinsic difficulty, that confines the topics to mathematical specialists, just as it did matrix algebra two decades ago. Obviously lack of knowledge and technique limits what can be done on an elementary level, but ordinal and cardinal operations on patterns, the basic characteristics of lattices (which have many every day interpretations), Boolean algebra, finite groups with patterns or permutations, and so on can be used to detect latent interest and talent.

I have said little about geometry. In a computing office geometry rarely appears explicitly, and outside it was true enough at one time to say that all one needed was the theorem of Pythagoras. This was because, with rare exceptions like skew gears, engineering was essentially two-dimensional; conceived on flat sheets and then folded up. Aircraft and ship construction were not quite so bad. The first certainly entails familiarity with changes of three-dimensional axes and rotating ones at that; the second with the description of curves by tangents and curvatures rather than by point co-ordinates. Also in such places as sheet metal and

pattern makers' shops one found an astounding implicit knowledge of ruled surfaces, edges of regression and the kinematics of folded sheets. With the coming of automatic machine tools whose "instructions" are worked out in computing labs. and which necessarily machine by tangential or osculatory methods, differential geometry and descriptive geometry may well come out of the relative shadow. The junior computor would do well to know what such things are about, if only to make the job look more real. This assumes that the computing office will be told something about the jobs that are given them. "Security" is often a legitimate excuse for not doing so; but equally often it is just a contraction for "Can't be bothered."

The one sure refuge from complete boredom when working on jobs that have no connection with anything, is to be interested in computing and computational entities for their own sakes. This is asking a lot of anyone but a born computor, but boredom will be mitigated if juniors know what sines, cosines, logarithms and so forth are, besides lists of numbers. (Incidentally, logarithms are used as mathematical functions, not as computing tools, these days.) It will be mitigated still more if personal skill at some mechanical device, desk machine, plotter or the like is not only permissible, but encouraged and recognised and not completely destroyed by the sight of seniors, and even supervisors, using these things in ways that would have meant instant dismissal if they had been working for, say, Dr. Comrie or the Nautical Almanac Office.

Teachers, it is up to you to improve things, for reform will have to start from the bottom up; there are not enough staff or field officers with battle experience. Insist on status and prospects for junior mathematical technicians (for that is what they are; not mathematicians and not clerks) comparable with other junior technicians, and insist that training should be done by those competent in training as well as in the topic. Finally, if the recruiting officer demands from girls and boys the combined qualifications of an academic mathematician and an experienced clerk in order to get them to sit in some administratively invisible den doing unexplained and tedious arithmetic for people who have no experience of controlling staff, tell him where he gets off.

A FURTHER USE FOR DRINKING STRAWS

Drinking straws provide a very useful multivalent aid when strung together on cotton. A "needle" may be made from a length of wire so that stringing is simplified. Tetrahedra are easily made by tying the straws in the appropriate shape. The octahedron requires two cotton ties across the diagonals of the square. A cube is a most irritating construction—never have twelve straws done so many things at the same time. Its rigidity is a very good study in equilibrium of frameworks. Any solid figure may be constructed in this manner including crystals.

Another use is to show the generators of ruled surfaces—a weak rubber band round a bundle twisted forms a hyperboloid.

We may also use them to study the rigidity of frameworks: having decided on a type of framework we may construct it using cotton as ties and straws as struts.

Their use in solid geometry is obvious.

C. HOPE.

SOLID PICTURES

T. J. FLETCHER.

The recent spate of stereoscopic films in the commercial cinema revived interest in other ways of producing stereoscopic pictures, and prominent among these is the anaglyph—the red and green pictures viewed through spectacles with filters of the same colours. Nowadays one associates this process with somewhat trivial publications, but the anaglyph has a long and distinguished history, and the early workers on stereoscopy were serious men of science who little knew that their inventions would produce the twentieth century space comic and the 3-D versions of the Lovely Ladies of the Charing Cross Road.

All stereoscopic systems depend upon the two eyes seeing the world around us from slightly different points of view, and the earliest recorded observations of the differences between monocular and binocular vision were made by Euclid. But no way of using two flat pictures to give different images to the two eyes was devised until Wheatstone made the first stereoscope in 1832. The form of stereoscope which is most common today was invented by Brewster twelve years later.

The idea of having the pictures in two complementary colours was first suggested by Helmholtz, and a practical system was constructed by Rollmann in 1853, but this used lantern slides projected on to a screen, and the familiar red and green printed picture did not appear until 1891. This was the invention of Ducos du Hauron who also coined the name *anaglyph* which came from the Greek, and meant an embossed ornament in low relief; a meaning which the word still carries in addition to the newer meaning which is nowadays more familiar.

This was the situation at the turn of the century. Whilst inventors continued to experiment with every conceivable type of system the first man to put the discoveries to practical use in teaching mathematics was Henri Richard, an educational pioneer whose name is almost forgotten. Richard was headmaster of the lycée Marceau at Chartres. Like so many teachers before and since he found that his pupils had difficulty in visualising three-dimensional situations, and so he prepared a number of three-dimensional illustrations in anaglyph. These were shown at the International Conference of Mathematicians at Cambridge in 1912 (although the official records contain no mention of them) and they were published by Vuibert, of Paris, under the title *Les Anaglyphes Géométriques* in the same year. This book is very scarce, but it is still possible to track copies down. After a sixteen page introduction and commentary there follow thirty-two diagrams on sixteen plates illustrating theorems on the volumes of solid figures, dihedral and polyhedral angles, cylinders, cones, descriptive geometry, crystallography and optical refraction. The introduction closes with the words, "We propose, M. Richard and I, to make little albums of anaglyphs for use in different branches of instruction. Furthermore, for secondary education we will make a collection of plates (principally of geometry and descriptive geometry) in a bigger format than this book. The suggestions of teachers on subjects which might be reproduced in anaglyph form will be valuable to us, and we will be pleased to receive them." I do not know if any of these further projects were ever carried out.

The next book of anaglyphs to appear came from Switzerland two years later. It was entitled *Le Relief en Géométrie* and was by Perregaux and Weber, professors at the Technicum in Le Locle. Switzerland is always a country of fine book production and this publication is the most handsome of its kind ever to be printed, with diagrams of fine quality and considerable complexity. There is a parallel text in French and German and there are fifty plates 15″ by 11″, half of which show lines and planes in three dimensions with theorems on perpendiculars and parallels, volumes, the five regular solids, spheres, cylinders, etc., while the rest are devoted to descriptive geometry. Descriptive geometry is a popular subject on the Continent, and the illustrations start with the fundamentals of projection on to the co-ordinate planes and rabattment, and proceed to the solution of more difficult constructional problems including drawing the intersection of a prism and a pyramid.

One may ask what advantages the drawings in these books have over models. The authors claim economy of space and cheapness of production. Both of these advantages must be conceded, but there are also disadvantages. In the first place the anaglyph process is an unsatisfactory one technically. Some people see these diagrams standing out with arresting clarity, and they can discuss complicated figures just as if they were really there in space before them, and they can describe geometrical configurations with which they were previously unfamiliar and which they have certainly never analysed before in terms which leave no doubt about the way in which the anaglyph is giving information to them. But others are not so fortunate and need time to get accustomed to viewing the drawings, and find that parts of the drawing which are farthest from the plane of the paper cannot be seen clearly because the two images will not fuse; and some people of course cannot use diagrams of this sort at all. The fate which these beautifully produced books have suffered and the oblivion in which they now rest indicates where the balance of opinion lies—in so far as anaglyphs are a substitute for space models they have been found wanting.

However anaglyphs demand consideration as teaching aids for other reasons, as was pointed out by Graf in some articles in the German educational press just before the last war. If an anaglyph or a stereoscopic film is seen from too close a viewpoint the picture is flattened, whereas if the viewpoint is too distant the picture is made deeper. If the observer moves to one side the picture does not behave as the scene would in reality, but objects are distorted by shearing. All of these distortions are affine transformations, and anaglyphs provide illustrations of affine geometry in a way that flat drawings or rigid models do not. (Affine geometry is the geometry which studies properties which are unaltered by parallel projection. It stands logically between Euclidean geometry and projective geometry.) These observations by Graf indicate the true status of anaglyph illustrations, and show how to put what might otherwise be regarded as drawbacks of the medium to didactic use. If stereoscopic mathematical films are made in the future it is to be hoped that their essentially affine character will be borne in mind; the makers should aim to make them about affine geometry rather than about Euclidean.

Sound as Graf's observations are it is not easy to see how they can best bear

fruit. The difficulty is to devise characteristically affine situations which exploit the peculiarities of the medium to advantage, and one sees the book *Mathematische Raumbilder* which Köhler, Graf and Calov published in Dresden in 1941 with mixed feelings. The subject matter is very much the same as in the earlier books, and a sense of the individual flavour of affine geometry does not emerge from the pictures as one has hoped it might on reading the excellent essay with which the book begins. The diagrams are masterpieces of draughtsmanship and one cannot fail to be impressed by the two drawings of Dandelin's spheres. The thirty-two plates are printed on a salmon pink background which makes viewing easier—an innovation due to Calov.

The same technique is used in the diagrams which Jack Coote has made to illustrate *Stereoscopic Transmission* by R. and N. Spottiswoode. These drawings are among the most effective which have been made by this process and the book is readily obtainable in England.

Completely new possibilities were opened up in America in 1940 when the Polaroid Corporation introduced the Vectograph process. In this the two drawings are reproduced photographically on two superimposed sheets of polaroid which are aligned with their directions of polarisation perpendicular. They are viewed in the usual way with the polaroid spectacles which have become familiar in the 3-D cinema. The effect is quite staggering, and all the deficiencies of the red and green system have been overcome. The figure leaps from the plane of the paper beyond all doubt, the eyes require no time to accommodate, and the light loss in the filters being much less the picture is bright and clear.

Little educational material has been made by this process; I can only trace a set of drawings illustrating Celestial Navigation which the Polaroid Corporation made for the United States armed forces during the war. The sample which I possess is "flatter" than it should be, but the explanation of this seems to be that the drawings were originally made to produce lantern slides. Pictures projected in this way are much larger than those on cards which are viewed directly, and parallaxes which are correct in the one case are quite wrong in the other.

Vectograph is expensive and it is not normally available in Britain, but it can be imported; and although it is not an easy material to use any mathematically inclined amateur photographer who wanted to learn the technique should be able to do so. Certainly there is a great need for someone to combine the best qualities of the work which has been done on the two sides of the Atlantic, and I have no doubt that if this was done stereoscopic illustrations would be produced of great interest and value. The whole field offers great scope for amateur research, because even those who do not want to go to the trouble and expense of importing Vectograph specially from America can still construct anaglyphs by the old process in coloured ink or crayon and investigate some of the many outstanding problems which they still present.

A little experiment will show that an ink such as Carnation Red gives excellent results when it is used for the red drawing, whilst Tropic Green is suitable for the complementary colour if it is diluted with many times its own quantity of water. Alternatively one can adopt the suggestions made by Mr. John Craig in the *Archi-*

tect's Journal of January, 1954, and use Derwent pencils, number 14 (red), number 40 (blue-green) and number 23 (purple). The purple shade is used for lines which overlap in the two figures and which are to be seen through both filters.

It is even possible to use suitably coloured chalks on a blackboard (provided that one reverses the colours!) but it is difficult to produce a stereo-drawing that would convince an impartial observer, and to deliver a lecture incorporating this technique would be quite an achievement.

Anyone who is sufficiently interested to undertake some of these experiments will no doubt derive pleasure in devising his own drawing methods. These are of two main types. One may calculate parallaxes algebraically or one may adopt modifications of standard draughtsmen's methods of drawing perspective views, using them twice over, once for each eye. The best of these is the one known as "the Architects' Method" which utilises intersecting projections from a side view and a plan. Its use in stereoscopic illustration has been described by Professor Rule in some papers in the *Journal of the Optical Society of America.*

The modest experimenter with inks and crayons will have one advantage over the more ambitious worker in Vectograph as he will be able to experiment on the introduction of colour as well. In October, 1955, *Picture Post* published anaglyphs in colour for the first time ever. These illustrations were all photographs, and they depend for their effect on the fact that the filters in the spectacles permit a sufficiently wide bandwidth to pass through to give the observer some sensations of colour. The two eyes, of course, see any particular object in the picture in two different colours, but they fuse these two colours to one when integrating their visual impressions.

Observers differ on how much colour they see in these *Picture Post* illustrations, and a lot of research remains to be done on this problem, but much could be done for a few pence. In any case the first public exhibition of geometrical anaglyphs in colour has still to be given, and we would be very pleased to stage it at some future A.T.A.M. demonstration.

FOOTNOTE.—An enquiry by Mr. I. Harris has revealed that the Polaroid Corporation are still only manufacturing Vectograph on an experimental scale, and so it cannot yet be obtained even with an Import Licence. We must hope that supplies will become available one day!

THE MATHEMATICAL MOUNTAINS

"ULYSSES."

IT is a long journey to the Mathematical Mountains. Starting from the north of Norway, you must travel to Spitsbergen, three days across the Arctic Ocean in a small boat. The way lies on from Longyear City and up Ice Fiord past the most northerly mining settlements in the world, and into Petunia Bay. You land at a small tumble-down shack by the shore; all around glaciers descend to the sea, and newly calved icebergs drift away south. You are only seven hundred miles from the North Pole, and henceforth you are on your own.

Starting in this way last year, our party carried stores, tents and sledges on our backs up a long valley, over the stony moraine and on to the glacier above. The first pull up was hard, steep and back-breaking; but once the ice began to level out the sledges ran more easily and we travelled in greater comfort. Our sledges were of a type designed by Nansen, and the technique of using them has changed little since the days of Shackleton and Scott. We had no dogs and pulled the sledges ourselves. Three men can manage about six hundred pounds. In the summer there is no night at these latitudes and you pull on until you are tired, pitch your tent, melt some snow for cooking, sleep, and then wake up to recommence the round; while high above the sun circles steadily, spiralling to all the quarters of the sky.

So we journeyed on; and as surveyor of the party I struggled with theodolites and sought to put a practical slant on my trigonometry. Before long we could see the ice-cap rising steadily to a line of rolling snow summits on the far horizon—these were the Mathematical Mountains. They do not as yet appear on any printed map, and some are still unclimbed. The highest of all, Mount Newton (or Newtontoppen as the name is in Norwegian), has been mapped for many years, although we never discovered how it originally got its name; but the mountains beyond remained unvisited until recently when parties from Cambridge University surveyed them for the first time. Following the associations evoked by "Newton," the Cambridge parties named the remaining peaks in the range after other mathematicians and astronomers.

So it came about that at the climax of our journey we skied northwards over a high pass and on down the mighty Veteranen Glacier with Eddingtonryggen on our left and the rocky summit of Galileotoppen on our right. The glaciers of Spitsbergen dwarf those in the Alps. Veteranen is over thirty miles long, and an airfield would seem lost on its vast expanse. The weather was perfect and we skied on in an ice-bound wilderness where few men had ever been before. Warm, golden sunshine flooded a landscape of azure and silver. Maclaurintoppen and Clairaulttoppen came upon our right to give way to Laplace and Galois as we moved along; Legendre and Poincaré were out of sight beyond.

It was a carefree day of bliss and relaxation, and the earth had never seemed a finer place. We did not say much to one another—there is little to be said when joy is perfect and one is back in the morning of the world.

We ate the food we had carried with us, and turned to follow our ski-tracks back the way we had come. There were grim moments on the journey home, with sickness in the party and delay through storm and blizzard, but the memory of the entrancing journey to the Mathematical Mountains will never leave my mind.

So when I lecture in the gloom of a London fog, "Newton" becomes a mountain, and the chalk dust is powdery rime glinting in the Arctic sun. A "Galois Field" is a field of eternal snow, and "Lagrange" is not an equation but the peak beyond and out of sight—a place we never reached. And as my students struggle with Maclaurin and Laplace, the memories merge to dreams of a longer journey to the empty land we had no time to visit—a wilderness no man has charted, a land that few have even seen; the land that lies beyond.

SOUTHAMPTON EXHIBITION

A one-day Conference and Exhibition took place at the Institute of Education of the University of Southampton on Saturday, November 19th, 1955. The arrangements were made jointly by the Association for Teaching Aids in Mathematics and the Institute. The Conference was attended by about three hundred people, some of whom had travelled a surprising distance, and it followed the general pattern of conferences held previously by the A.T.A.M. in other parts of the country.

The first three lectures were given on the central theme of *Teaching and Learning Aids*. Dr. C. Gattegno described the purpose of the Conference and introduced the remaining speakers. He then discussed teaching aids for use in Primary Schools and stressed in particular, the great potentialities of the material which had been developed in Belgium by M. Cuisenaire.

He was followed by Miss Y. Giuseppi who described the "Live Material" which she had used successfully in Modern Schools. Menu cards, timetables, recipes, dress-patterns, etc., all provide mathematical material which weaker pupils find more stimulating than the questions from a book. Much of the arithmetic in schools was archaic, and the time could be much better employed doing suitable geometry involving the study of shapes and patterns.

In the afternoon Mr. R. H. Collins spoke on Grammar and Technical School work. Challenging much established practice he exhorted his audience to seek methods of proof and demonstration which were aimed at convincing boys rather than accomplished mathematicians—and not merely clever boys but the less able boys as well. He showed bar linkages which demonstrate geometrical theorems, models illustrating area properties, two of his own filmstrips and a Nicolet film, and must have astonished many present by his plans to introduce calculus in the middle forms of a Technical School.

Mr. T. J. Fletcher's subject was "Mathematical Films: their Place in Mathematical Education." After showing geometrical films by Nicolet, Cantagrel and himself he compared their various approaches and considered how they could best be fitted into a course of teaching. He outlined his plans to promote film production by teachers themselves and invited those present to collaborate.

All day during gaps between lectures visitors toured the Exhibition of models and teaching apparatus under the supervision of Mr. J. Peskett. This exhibition continues to display fresh ideas at each appearance, and this time beside the old favourites of solid models and machines for demonstrating the fundamentals of mechanics we found an ingenious new device illustrating Fourier series. Another room contained an exhibition of filmstrips.

The thanks of the Association must go to Dr. K. M. Lobb and Dr. Evans of the Institute for making the local arrangements and for making the visitors feel so very much at home.

There was only one note of regret. All of the speakers gave formal lectures, and there were no demonstrations with a class of children. Visitors may well be sceptical when this is the case since the crucial test of teaching theories is how they

stand up in front of a class. Whilst the demonstration lessons which we have arranged in other parts of the country may have left something to be desired in their attempts to simulate normal classroom conditions the Association is convinced that the effort is well worth making, and it was a disappointment that this time the Local Authority could not see a way to provide classes of children for our teachers to teach.

BRISTOL

The Association staged an Exhibition and Demonstration at Bristol on February 4th, 1956. We were fortunate in having the use of Hengrove School, a most attractive new building standing in spacious grounds on the outskirts of the city. Demonstration lessons were given by Dr. Gattegno using the Cuisenaire number material, Mr. C. Hope, Mr. R. H. Collins with filmstrips, and Mr. I. Harris with a film. Mr. J. Peskett gave a demonstration lecture of materials which can be used for manufacturing models and apparatus as well as arranging the comprehensive exhibition. The usual display of films and filmstrips took place in another room. The Annual General Meeting of the Association was held earlier in the day.

In conjunction with this demonstration a week-end seminar was arranged at Shipham, a few miles outside Bristol, on the theme *From Teaching Aids to Formal Proof*. It was discouraging for the organisers to find that, in contrast to the day exhibition in the city, the seminar was very poorly supported. In view of the unfavourable time of the year and the rather inaccessible place in which it was held, this is perhaps understandable, but the Committee will have to give very careful thought to the planning of similar events in the future.

The Editorial Board is anxious to encourage a high standard of illustration in this journal, and some teachers of technical drawing might be willing to give assistance in this. The work required is chiefly tracing or redrawing rough sketches in a form suitable for the preparation of blocks by the printers. Anyone prepared to help in this way should write to the Editor.

Mr. R. H. Collins addressed a meeting of the Mathematics and Science Section of the Scottish Secondary Teachers' Association at the High School for Girls, Aberdeen, on Saturday February 11th, 1956. His subject was *Learning Aids in Mathematics*, and his talk, which was extremely well attended, was illustrated by an exhibition of teaching aids many of which he had constructed himself.

We have very great pleasure in announcing that the Nuffield Foundation have offered a most generous grant to the Association Film Unit. The grant is for the purchase of an animation camera and some associated equipment, and is particularly intended to encourage the development of three-dimensional mathematical films.

TEACHING OF MATHEMATICS

The Ministry of Education and the Kent Education Committee arranged a week-end course in March, which examined the *continuity* of the Teaching of Mathematics from the Primary School to the University. An opening talk on "The Background to the Teaching of Mathematics in the Twentieth Century" (P. C. Davey, H.M.I.), set the high standard which was maintained throughout the course.

The programme included sessions on: "Mathematical Ideas in the Primary School," introduced by J. T. Browne (Headmaster); "Mathematics in the Secondary Grammar School," introduced by E. Barnes (Headmistress), and R. L. Hudson (Headmaster); "Mathematics in the Secondary Modern School," introduced by J. Briddon (Headmistress), and C. H. Sprawls (Headmaster).

All agreed that it had been instructive and helpful to be made acquainted with the problems and difficulties of our colleagues in the profession.

It was recommended that local consultation between schools would be the most valuable contribution towards continuity, and it was thought that the initiative should come from the Primary Schools. The standardisation of method and content of the syllabus being important items for such meetings.

The Primary Schools should concentrate on giving a fuller understanding, with a horizontal rather than a vertical spread of knowledge, with no over-direction towards the eleven-plus examination.

The content of this examination should have a bias towards non-trainable material and include problems which test the child's understanding and not solely his attainment in computation.

The Grammar School should pay more attention to the welfare of its new entrants, who are probably meeting real competition for the first time in their new school. They must avoid the dangers of over-specialisation.

The Modern Schools should first gain the confidence of their pupils after their recent "failure" and should avoid direct recapitulation of work already done in the Primary School. Later, with explorations into new topics, encourage pupils to have a positive attitude in mathematics.

Mathematics has been said to be "the knowledge of all dark things," we hope that our deliberations have thrown some light on to this.

IAN HARRIS.

A MATTER OF LIFE AND DEATH

A man is in prison. The prison has two doors, and he is told that he may leave by either, but one is the door to freedom and the other is the door to death. He is allowed to put one question to each of two slaves who wait on him and who know the secret of the doors. Now the prisoner is aware that one of these slaves always speaks the truth and the other always lies, but he does not know which is which. What question can he frame so that he may learn the way to freedom?

Solution on p. 32.

BOOK REVIEWS

General Elementary Mathematics. Hodges & Wood (Blackie.) Part I, pp. 279; Part II, pp. 285; Part III, pp. 346; Part IV, pp. 326.

This is a Four Year Course intended for abler pupils up to General Certificate Ordinary Level. Each part is intended to be one year's work. It is an unusual book in that it appears to be planned on the lines of continental text-books rather than in the British tradition. A topic is introduced and treated exhaustively in a single chapter. For example, logarithms are dealt with in the third year; this chapter begins with the elements and goes on to cover not only multiplication, division, roots and problems, but also the more general work usually found only in the Additional Mathematics syllabus. This arrangement postpones the pupil's introduction to many topics and tends to make his mathematical progress a series of leaps rather than a steadily developing continuous course.

In the chapters on Geometry it seems to the reviewer that too many theorems are proved (in all about forty) and in this respect the book fails to live up to the authors' claim to have got away from "detailed knowledge of formal Geometry."

Much of the Algebra and Arithmetic is well done and neatly integrated. A chapter on Ratio and Proportion covers both Algebraic and Geometric work. It is doubtful, however, whether the proof that $a/b=c/d=(ma+nc)/(mb+nd)$ will find much response in even the abler pupils if introduced, as here, at the beginning of the third year's work. The chapter on Directed Numbers comes early, as it should, but the authors have nothing new to say here. We are once again asked to consider "a car with its nose pointing west with the gear lever in reverse($-$) the car will travel east $(+):(-)\times(-)=(+)$": an argument which forever fails to convince.

In Part IV two chapters on the Calculus do not quite cover syllabuses for Additional Maths. Applications to kinematics, turning values, areas and volumes are dealt with, but there is no mention of rates of change or the method of small increases, except for a few examples and these require knowledge of work not covered.

The general layout is rather stolid and unexciting, though the diagrams are clear and adequate. There is an abundance of good examples with revision exercises and test papers at the end of each part.

This is a book not for the abler pupils but only for the ablest.

R. D. KNIGHT.

Numbers in Colour, a new approach to the teaching of Arithmetic. By G. Cuisenaire and C. Gattegno. Pp. ix, 56. Price 5s. (Heinemann.)

This booklet, the second edition of one first published in 1954, differs from its predecessor in that two new chapters have been added to deal with new findings since the first edition was written.

The two authors have divergent views on the teaching of arithmetic, which are set out separately to explain how Cuisenaire material may contribute to an understanding of arithmetic. Without some knowledge of the material, many parts of the book are incomprehensible; with the material, the reader will be stimulated to try out many things for himself.

The idea of different sized blocks, each multiples of one unit, is not new. It has been claimed that H. G. Wells suggested it at the turn of the century, but the device is probably much older. The Cuisenaire material consists of rods or blocks each 1 cm. square in section and of integral lengths. Its novelty is that it relates the different families of integers by families of colours. For example, the crimson, vermillion and brown rods each contain red, and may represent 2, 4, 8 respectively.

Cuisenaire, a Belgian teacher, uses the material to teach the requirements of the state syllabus for the different age groups. He takes the view that only when calculations have become automatic, is the mind free to enter other fields, in particular, reason. He teaches all four processes and fractions simultaneously. Specimen lessons are given to illustrate the stages of development from the beginnings to the solution of conventional problems. His treatment of multiplication facts deserves special mention. A teacher will find a method ready to hand here.

Gattegno starts from the premise that the symbols of arithmetic are as much an aid as the materials themselves to the understanding of arithmetic. Symbols describe mental activity which is concerned with material in the early stages. His section contains many provoking ideas, e.g. fraction as the inverse of multiplication. He strives in a most stimulating way to bring primary school mathematics into line with modern thought. His final section on his experiences with children is concerned with the positioning of rods to illustrate various processes, consideration of long division and use of number charts. This section is extremely valuable and interesting.

"The reader should try to discover not how the rods can illustrate what he knows, but rather what the material will enable him to do." This is the key to Gattegno's teaching method. Whether the new method will work is a matter for experiment to decide. Its claims are sweeping; in many cases "some children" would be more accurate than "children." One wonders, in spite of the assurances of the preface, whether the approach to mathematics is too self-contained and isolated from other social and scientific mathematical experiences.

The pamphlet contains many ideas which will be new to the majority of primary school teachers. Many will see mathematics in a new light, all might profit from some sections but many will find Dr. Gattegno's section rather brief and lacking in teaching detail to make their use of the material fully effective. Nevertheless, most teachers will find their approach to arithmetic enlivened after reading the book in conjunction with the material.

C. H.

A Note Book in Applied Mathematics, by L. Harwood Clarke, M.A. (William Heinemann Ltd.). 9s. 6d.

This is a welcome companion to the author's *Note Book in Pure Mathematics* and covers the work necessary for Advanced Level G.C.E. but not for Scholarship Level.

It is in effect, a very condensed textbook, divided into three sections, Statics, Dynamics, and Hydrostatics. Each section begins with a summary of definitions

and ends with a small selection of questions taken from the past examinations of several Examination Boards. It is well set out, the diagrams are well drawn and clear. Typical examples, easy to follow, are worked for the student, and there are plenty of straight-forward exercises for the student to work for himself.

Two minor criticisms are (a) it does not emphasise sufficiently that the Velocity Ratio of a Machine depends only on the "geometry" of the Machine; (b) it seems pointless to state "A mass of one pound when at the *Equator* weighs one pound weight." However, the idea of showing examples in Dynamics worked, in parallel columns, in both Gravitational and Absolute Units is a very good one.

This book should prove to be very useful indeed for revision purposes.

J. W. PESKETT.

The Principles of Arithmetic by A. Keith and J. Robertson, Blackie & Son, London, 1955 (first edition 1940). 8s. 6d.

I read this book with continuous interest. The agreeable and rigorous style in which it is written could be a model for teachers of mathematics. Prospective teachers who study this volume will find much opportunity for learning about mathematics, its history and its teaching. It is clear that the authors have very perceptive minds and wide experience, and their appreciation of traditional methods is tempered by a critical attitude and a knowledge of realities. They give the impression of having followed each idea to where it obviously leads, and of accepting responsibility for departing from the usual practice when it is contrary to common sense or to the findings of research. On most grounds it is a commendable book.

Instead of entering into details as to the many valuable and enjoyable parts of the book, I should like to say a few words about the thoughts that formed themselves in my mind each time I found my experience differing from that of the authors.

In the first place it seemed to me that counting occupied too big a place as the background of arithmetic. This is of course so in the case of most teachers, but since as early as 1940 Piaget attempted to influence teaching by showing that number is not a primitive notion and not the result of counting, but the outcome of many complex activities, one would have liked to see counting given its true place.

Secondly, the sub-division into chapters dealing separately with the four operations appeared to be not merely a convenient device for making the work more easily readable, but rather to represent the pattern of the authors' thought. What is said, however, in the chapter on fractions, shows that the four operations can be studied at the same time, and this is, to my mind, a more realistic and profitable line of approach in number work.

Thirdly, it was clear that the thinking of the authors was dominated by the discontinuity of the material used for the formation of number, for the introduction of operations and their stages of development. When they pass on to fractions, and use continuous and discontinuous materials, they propose a more comprehensive scheme. But even here the interference of established habits

(proper and improper fractions, for instance) seems to me to cause them to miss the opportunity of giving a more truly mathematical bias to their text.

Lastly, for this 1955 reprint, the bibliography could have been brought up to date, for it is likely that students in training colleges who study this book will rely for further reading on what the bibliography contains.

I hope that our readers, at whatever level they teach, will enjoy the experience of reading this book. That they will find in it food for thought I have no doubt.

C. GATTEGNO.

Fundamental Number Teaching. By R. K. & M. I. R. Polkinghorne. Pp. 196. (Harrap, 1955.)

The authors have been writing for many years in *Teachers' World*, and this book is the response to a request from 2,000 correspondents for a helpful book.

With the modern emphasis on activity and group work, it comes almost as a breath from another age. The authors clearly regard the good teacher as one who carefully analyses the material to be presented so that each difficulty may be explained before it is met. The book is a most thorough and painstaking survey of this approach to the teaching of arithmetical computation. The reader is led through addition, subtraction, multiplication, division, compound quantities, fractions and decimal fractions. Each step in the teaching sequence is thoroughly and carefully explained. The most valuable features of the book are the grading of examples and the comprehensive collection of drill-devices and games; for these alone the book is worthy of study.

The authors make no claim to a scholarly treatise on arithmetic. Perhaps because of this, the book does not give an impression of mathematics as continuously developing throughout the pupil's school life. Mathematics is seen as a long sequence of correct exercises neatly set out in a book. Understanding by the pupil is aimed at by demonstrations; always a demonstration to convince the pupil (or the class) of a procedure, result or relationship presented by the teacher.

The teacher who is looking for guidance in presenting mathematics according to post-war ideas with experimental approaches, discoveries and (mental) activity will find little help here; the teacher who has gone some way along the road and requires help with drill will find this a most useful book, but the teacher who is unsure of his method, uncertain of his own arithmetic, will find security for himself and some success for his children by following the author's ideas.

Comprehensive Mathematics. By J. D. Hodson. MacMillan. (With and without answers. Price 8s. 6d.)

There are no frills about this book, it concentrates on presenting a thorough revision course in Elementary Mathematics for Fifth Forms, in preparation for the General Certificate of Education (Ordinary Level) Examination, the bias being directed towards the Alternative Syllabus.

The order of introduction of the various topics is good, and interest is maintained throughout. There is a continual revision of previous work by Test Papers, at the end of each chapter; and by the re-introduction of material at advancing levels of difficulty.

The constant change and contrast of topics is refreshing after the stilted artificialities of over-unification to be found today. The true unification of Mathematics comes from the teacher rather than from a book. The varied contents of Chapter 21: Construction of Circles, More Simplifications and Equations, Relative Velocity, Graphs of Sine and Cosine, and a Test Paper—all in eight pages, is typical of the other thirty-four chapters.

The last three chapters provide a formal course on Elementary Differential Calculus, Elementary Integral Calculus and Elementary Co-ordinate Geometry; all of which have been introduced incidentally through the book. This final section would be of greater value with more examples.

The book caters for the range of ability expected in a Grammar School, but needs a scattering of "starred" difficult examples and test papers for the very intelligent pupil.

The typography is admirable and the quality of the diagrams is very good, as is the clearness of expression of the author.

Mathematic's books may be divided into the two classes, "text-books" and "books of examples." Comprehensive Mathematics deservedly earns the title text-book. Well done, Mr. Hodson!

IAN HARRIS.

BOOKS RECEIVED FOR REVIEW

Title	*Author*	*Publisher*
1. The Principles of Arithmetic	Keith & Robertson	Blackie
2. General Elementary Mathematics (4 vols.)	Hodge, E. D., & Wood, B. G. J.	Blackie
3. Comprehensive Maths.	Hodson, J. D.	MacMillan
4. Practical Arithmetic for Girls	Harris, R. E.	MacMillan
5. From Simple Numbers to Calculus	Colerus, E.	Heinemann
6. A Notebook in Applied Maths.	Harwood Clark, L.	Heinemann
7. Numbers in Colour	Cuisenaire, G., & Gattegno, C.	Heinemann
8. The Theory of Numbers	Jones, B. W.	Constable

In the earlier inscriptions of India dates and other numerals are written in a notation not unlike that of the Romans, Greeks and Hebrews, with separate symbols for the tens and hundreds. The earliest inscription recording the date by a system of nine digits and a zero, with place notation for the tens and hundreds, comes from Gujarat, and is dated A.D. 695. By this time, however, the new system had been heard of in Syria, and was being used as far afield as Indo-China. Evidently the system was in use among mathematicians some centuries before it was employed in inscriptions, the scribes of which tended to be conservative in their system of recording dates; in modern Europe the cumbrous Roman system is still sometimes used for the same purpose. The name of the mathematician who devised the simplified system of writing numerals is unknown, but the earliest surviving mathematical texts—the anonymous "Bakshali Manuscript," which is a copy of a text of the 4th century A.D., and the terse *Aryabhatiya*, written in A.D. 499—presuppose it.

A. L. BASHAM, *The Wonder that was India*.

FILMSTRIPS

The following filmstrips have been received with our thanks:

	s.	d.
Educational Productions Ltd.		
Junior Mathematics—		
(1) The Hen Who Couldn't Count	15	0
(2) Count 'Em Cowboy	15	0
(3) There Was An Old Woman Who Lived in a Shoe	15	0
Mathematics—		
(1) The Bicycle	15	0
(2) The Football Field	15	0
(3) Chess	15	0
Treasures of the Triangle—		
(1) Concurrent Lines O and I	15	0
(2) Concurrent Lines G and H	15	0
(3) Collinear Points	15	0
Hulton Press Ltd.		
The Five Regular Solids	9	0
The Use of Graphs—		
(1) Mathematical Pictures	12	6
(2) Graphs of Relationship	12	6
(3) The Graph as a Moving Picture	12	6
(4) Measuring in Both Directions	12	6
Mathematical Pie Filmstrips		
The History of Calculation	15	0
History of Numerals (Part I)	15	0
An Introduction to Loci	15.	0
An Introduction to the Slide Rule	15	0
An Introduction to the Calculus	15	0
The Quadratic Graph	10	0
From Experiment to Law	10	0
The Graphical Solution of Equations	17	6
An Area Construction (colour)	30	0
Permutations	17	6
How to Read Your Tables	12	6
Unicorn Head Visual Aids Ltd.		
Fractions are the Parts of a Unit	15	0
An Introduction to Geometry	5	0

All these Filmstrips may be seen at our Exhibitions, and it is hoped to build up a comprehensive collection of all Mathematical Filmstrips obtainable. Reviews of new and recent issues will be included in future bulletins, and a detailed catalogue of all such material is being compiled.

It would be appreciated if readers would send in to me details of any Filmstrips on Mathematics, particularly those not widely known in this country, for instance, foreign or private ones.

IAN HARRIS,
Grammar School,
Dartford, Kent.

FILMS.

NON-EUCLIDEAN GEOMETRY. Produced by the D.E.F.A. studio under the technical supervision of the Mathematics Department of the Humboldt University, East Berlin. Mathematical Director, Professor Kurt Schröder; Scenario and technical direction by Dr. Hans Kaiser; Animation, Hugo Cernik; Camera, Hannes Quauke. Distributed by the Zentralinstitut für Lehrmittel, East Berlin. Black and white, silent, in three parts totalling 27 minutes. 16mm.

The production of *Non-Euclidean Geometry* is a landmark in the history of the mathematical film, since it lasts for nearly half an hour and the mathematics which appears in it is the most advanced ever to be filmed so far. Movements of the hyperbolic plane over itself, and the applications of hyperbolic geometry to the theory of functions are topics with only a very limited appeal, and so it is all the more noteworthy that so much thought and care have been devoted to a film about them. Since its subject matter is specialised this film will never become widely popular, but it should be studied not only by all those who teach advanced geometry but also by all who are interested in the design and manufacture of mathematical films at whatever level of difficulty.

This film is intended for lecture illustration, and so it makes no attempt either to give a complete treatment of non-Euclidean geometry from the fundamentals onwards or to prove basic results which are better proved by algebra during a lecture. The aim is to use the film to give extra illustrations and to carry on the exposition when the limits of traditional methods are reached. When a film is made for university use this is the correct approach. As part of the scheme a small explanatory booklet is supplied which gives a synopsis of the scenario and a long algebraic introduction to the less familiar ideas which are involved. The suggestion is made that since the group of hyperbolic movements is isomorphic with the group of bilinear transformations of a complex variable the geometry in the film is important not only for its intrinsic interest but also because of its applications to the theory of functions. In other words the film can provide a conceptual background to a range of problems in other parts of mathematics.

A non-Euclidean movement is a collineation which leaves the metric conic (in this case an ellipse) invariant. Movements in hyperbolic geometry are of three types according to whether the movement has a fixed point outside, inside or on the metric conic. The first case has no Euclidean analogue, the second is analogous to an ordinary rotation and the last to an ordinary translation. The three parts of the film illustrate these three cases, and the action develops in the same way in each. This emphasises the points in an effective manner, and seeing all the ideas three times over with slight variations fixes them in the memory.

Each section shows the inter-relations of five mappings of the hyperbolic plane and the trajectories of a movement which is being studied. First the hyperbolic plane is mapped in the inside of an ellipse (the Cayley or Klein model), and this ellipse is transformed to a circle by a parallel projection The circle is projected in turn on to a hemisphere bounded by it, and the hemisphere then receives two different stereographic transformations; the first from a point on its boundary gives a mapping referred to as "the complex plane" (the reasons for this are not obvious

in the film but are fully explained in the booklet) and the second, from its pole, gives the well-known Poincaré model. At each stage a grid of points is projected and these are joined up by the trajectories of the movement as the plane slides over itself.

The drawing of the film presents interesting problems because most of the action is in three dimensions. The three dimensional projections with the use of air brushing and other clever suggestions of perspective receive high praise from all who see them, and the form of presentation adopted makes the relationships between the different maps extremely clear. The indication of the actual movements is perhaps less successful, and one cannot help feeling a little disappointment that some opportunities have been missed. The movements of the plane over itself might have been conveyed more clearly by animated patterns of the type drawn by Motard in the French films. If these had been combined with the drawing-in technique actually employed not only would the movement have been built up throughout one cycle but it would have continued indefinitely. At the same time a striking artistic pattern would have emerged at each stage and the extra sequence using a little triangle to show the movement of an element of area across the diagram, which must have been rather a nuisance to film, would not have been necessary.

The climax of a mathematical film should contain the most exciting movements, and to build up to a diagram which "sticks" at the critical moment introduces an unwelcome anti-climax. The artistic aspects of these films always deserve the closest attention because the most beautiful solution of the artistic problems nearly always leads to the most lucid exposition of the mathematics. But these criticisms are made in a friendly spirit and the Humboldt University are to be congratulated on a most courageous enterprise. This film will become a classic and we hope to see more from them in the future.

Non-Euclidean Geometry has been acquired by the British Film Institute for the special library which they hold on behalf of the British Universities Film Council, and is available from them on loan. An English translation of the notes which accompany the film can be obtained on application to me, at Sir John Cass College, Jewry Street, London, E.C.3.

T. J. FLETCHER

SOLUTION TO PROBLEM.—It is all a matter of the multiplication rule of signs. A question that will do is, "If I should ask you 'Is this the door to freedom?' would your reply be 'Yes'?" He asks this pointing to one of the doors, and if it is the right one both slaves will reply "Yes," and if it is the wrong one both will reply "No." The condition that one slave always tells the truth while the other always lies is unnecessary with this solution, it is only necessary that each should be consistent in his behaviour.

Another question which does demand that one slave should tell the truth and one lie is, "Would the other slave reply 'Yes' if I asked 'Is this the door to freedom?'" To this question both slaves would reply "No" if it was the door to freedom, and "Yes" if it was not.

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Contributions to the journal should be sent to the Executive Editor. Contributors are asked to keep mathematical notation as typographically simple as they can, and to submit illustrations, where necessary, in a form suitable for reproduction.